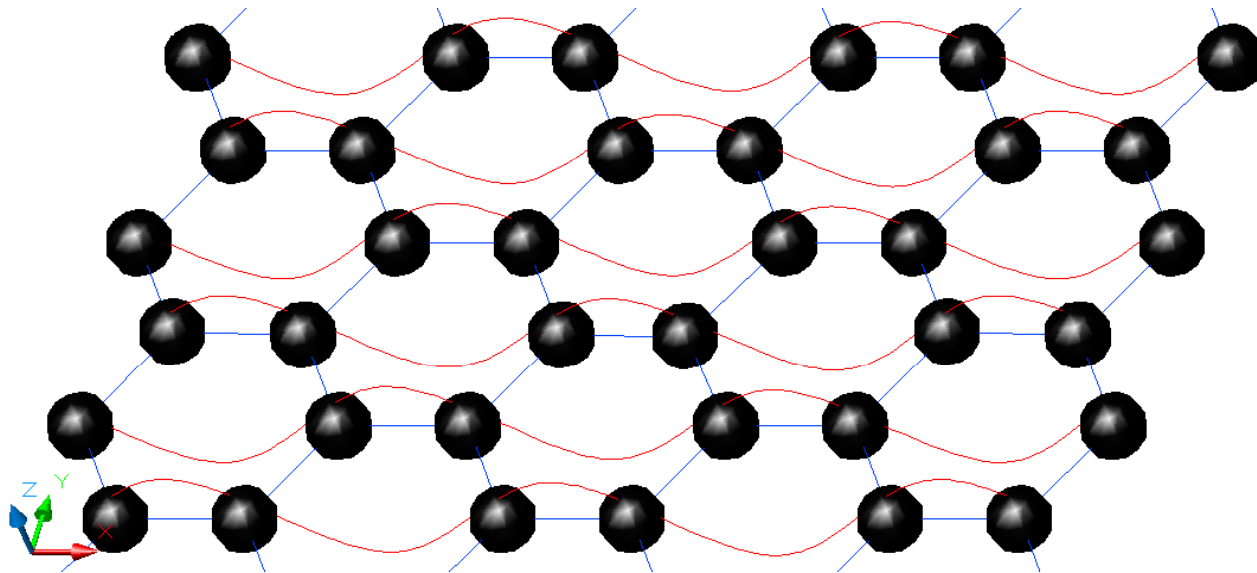
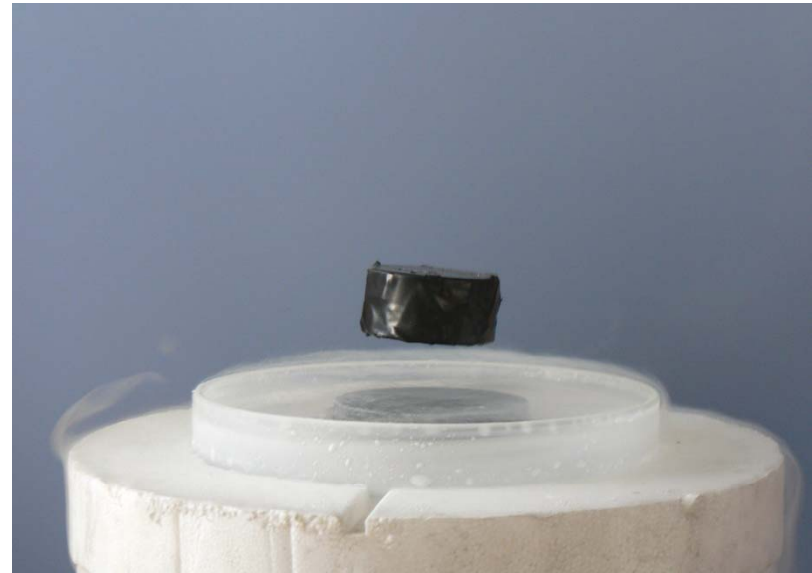


# Basic Properties of Charge Density Waves



# Interesting Properties

- Non-linear DC and AC Conductivity
- Metastable Memory
- Metal-Insulator Transition
- “Superconductivity”
- ...



# Overview

- Formation of CDWs in one-dimensional metals and extension to higher dimensionality
- Dynamics of CDWs
  - Effect on electrical transport

# Peierls Instability

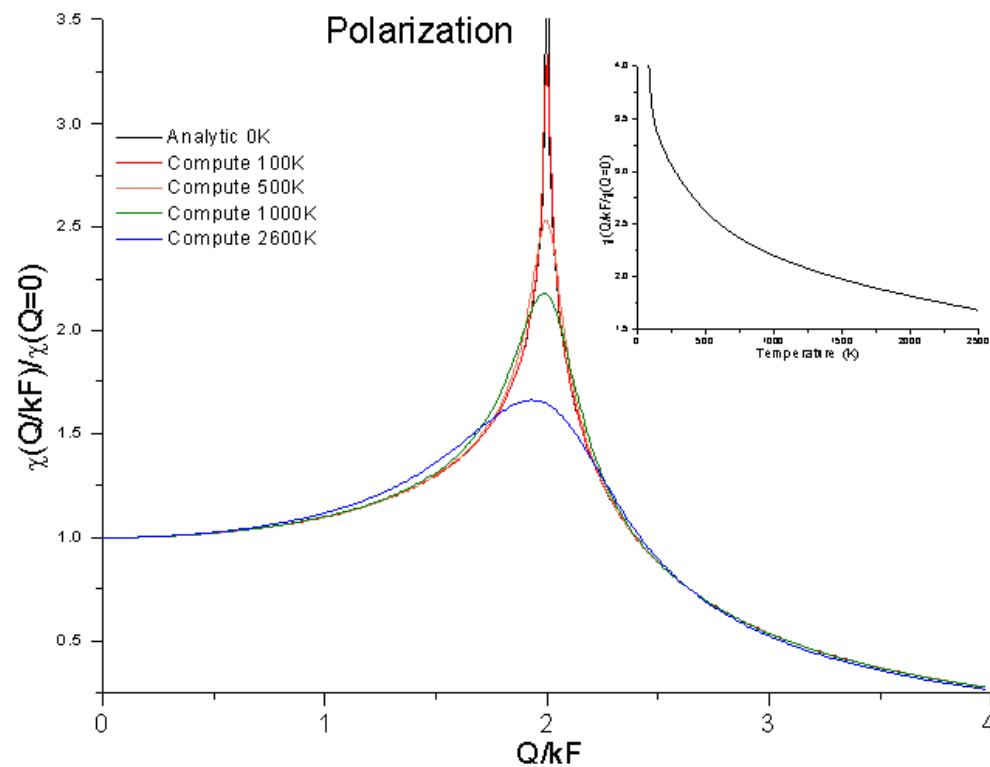
Applying an oscillating potential with wavelength  $2\pi/Q$  to a one-dimensional free electron model;

$$V(r) = V_Q e^{iQr}$$

$$\rho(r) = \frac{1}{N} \sum_k |\psi_k(r)|^2$$

$$\cong \rho_0 + \delta\rho(r)$$

$$\delta\rho(r) = \frac{1}{V} \sum_Q V_Q \chi(Q) e^{iQr}$$



# Kohn Anomaly

TABLE II: Top-10 cited PR articles. The asterisks denote citation undercount due to citations with missing prepended A/B page numbers – 123 out of 3227 total for item 1 and 120 out of 2640 for item 2.

Cite Rank	Publication			# cites	Av. Age	Impact	Title		Author(s)
1	PR	140	A1133	1965	3227*	26.64	85972	Self-Consistent Equations...	W. Kohn & L. J. Sham
2	PR	136	B864	1964	2460*	28.70	70604	Inhomogeneous Electron Gas	P. Hohenberg & W. Kohn
3	PRB	23	5048	1981	2079	14.38	29896	Self-Interaction Correction to...	J. P. Perdew & A. Zunger
4	PRL	45	566	1980	1781	15.42	27463	Ground State of the Electron ...	D. M. Ceperley & B. J. Alder
5	PR	108	1175	1957	1364	20.18	27526	Theory of Superconductivity	J. Bardeen, L. N. Cooper, & J. R. Schrieffer
6	PRL	19	1264	1967	1306	15.46	20191	A Model of Leptons	S. Weinberg
7	PRB	12	3060	1975	1259	18.35	23103	Linear Methods in Band Theory	O. K. Andersen
8	PR	124	1866	1961	1178	27.97	32949	Effects of Configuration...	U. Fano
8	RMP	57	287	1985	1055	9.17	9674	Disordered Electronic Systems	P. A. Lee & T. V. Ramakrishnan
9	RMP	54	437	1982	1045	10.82	11307	Electronic Properties of...	T. Ando, A. B. Fowler, & F. Stern
10	PRB	13	5188	1976	1023	20.75	21227	Special Points for Brillouin-...	H. J. Monkhorst & J. D. Pack

Redner, S. [arXiv:physics/0407137v2](https://arxiv.org/abs/physics/0407137v2) [physics.soc-ph]

# Kohn Anomaly

Fröhlich Hamiltonian for non-interacting electrons in the one-dimensional jellium model

$$H = \sum_k E_k a_k^\dagger a_k + \sum_Q \hbar \omega_Q b_Q^\dagger b_Q + \frac{1}{\sqrt{V}} \sum_{k,Q} g_Q a_{k+Q}^\dagger a_k (b_Q + b_{-Q}^\dagger)$$

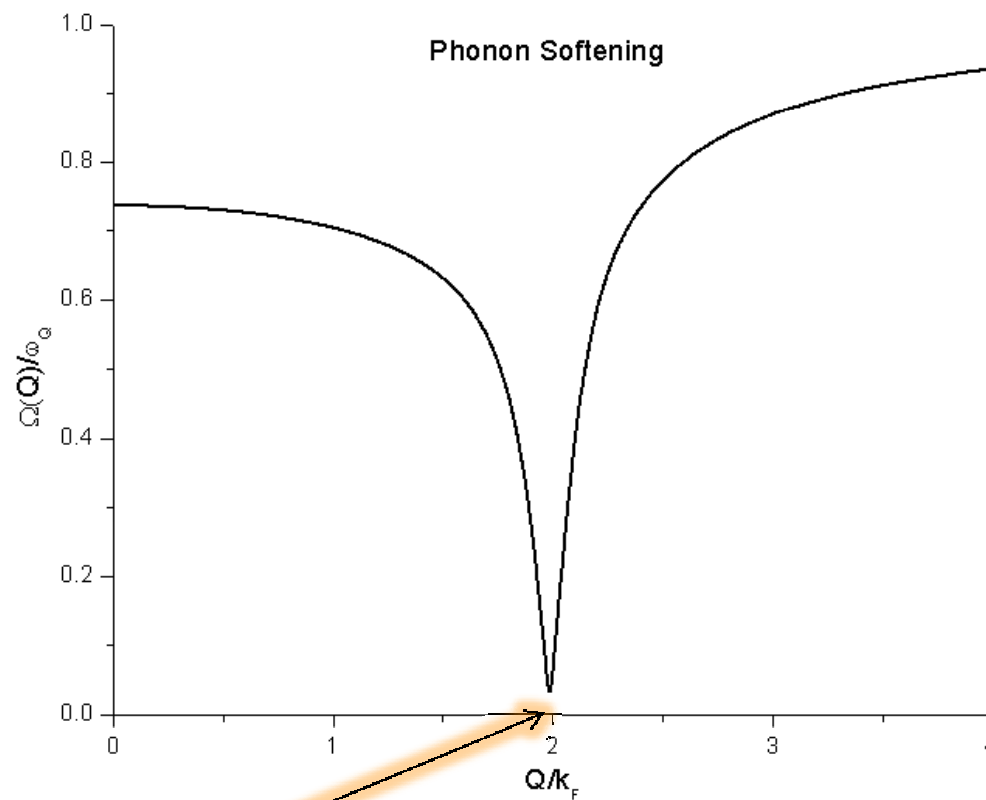
Normalized acceleration of Fourier components for lattice distortion

$$\Omega_Q^2 = -\frac{\ddot{n}_Q}{n_Q} = \frac{[[n_Q, H], H]}{\hbar^2 n_Q} = \omega_Q^2 - \frac{2}{\hbar} |g_Q|^2 \eta \omega_Q \chi(Q)$$

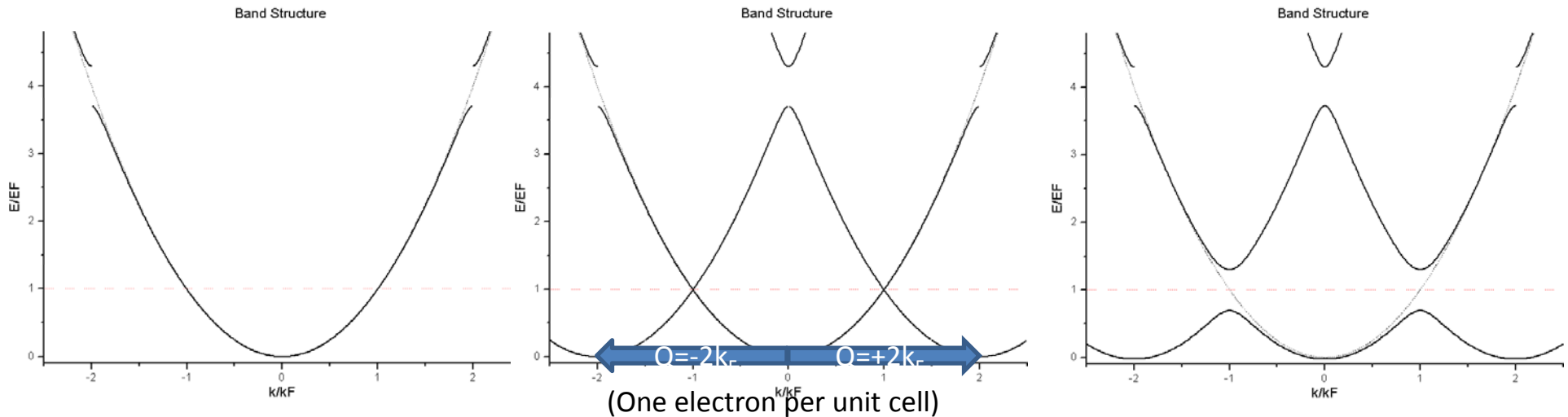
Electrons screen the restoring force of ion distortion, thereby reducing the frequency of phonons.

# Kohn Anomaly

Electrons screen the restoring force of ion distortion, thereby reducing the frequency of phonons.



# Band Structure



Change in Strain Energy due to a lattice distortion;

$$\Delta U = \frac{c}{2} u_Q^2$$

Change in electron energy due to band gap;

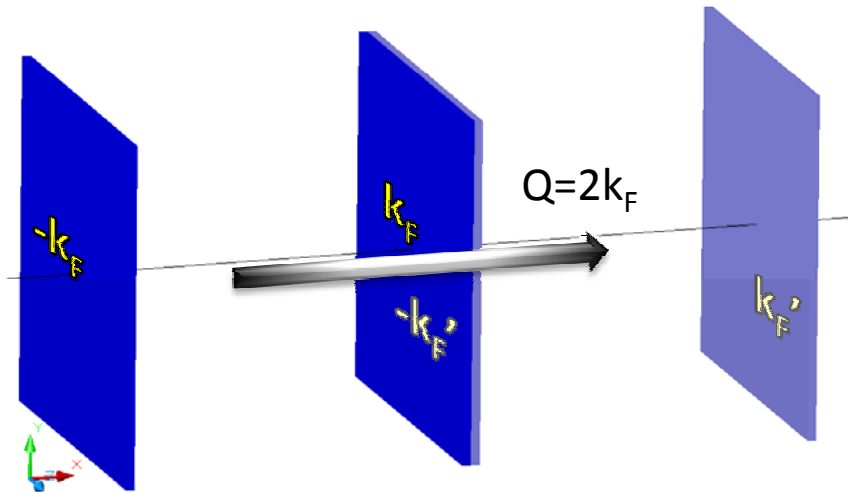
$$\Delta K \simeq -|g u_Q|^2 N \chi(Q)$$

If total energy is reduced, CDW spontaneously forms.

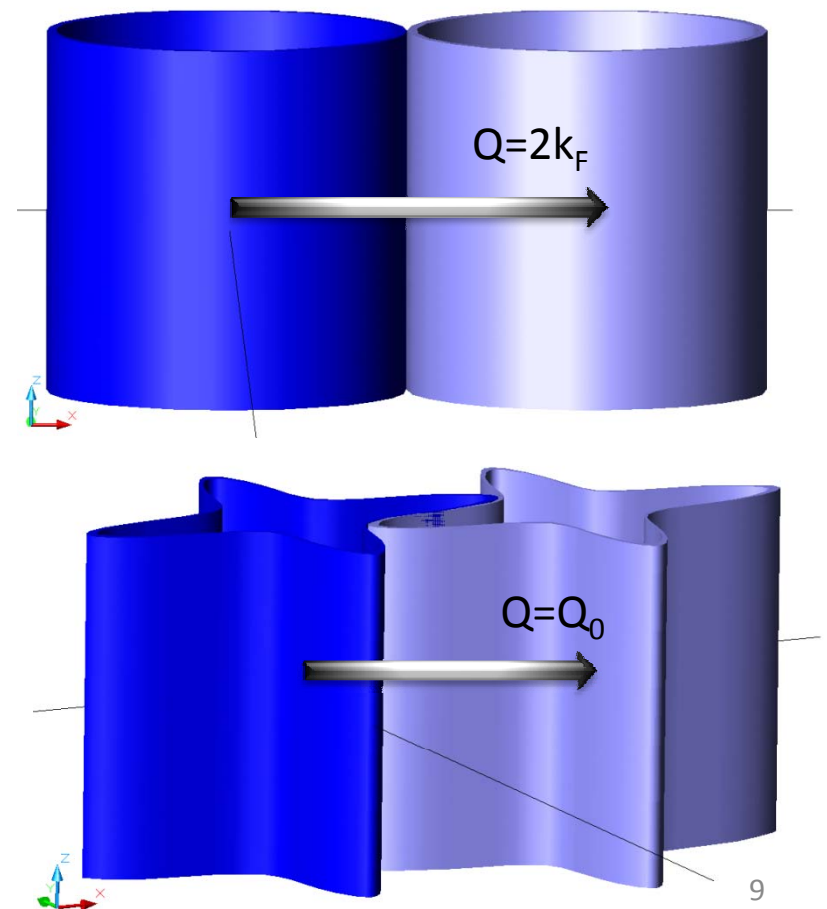


# Nesting

The singularity in the Polarization function is the result of overlapping Fermi surfaces when the original Fermi surface is displaced by a wavevector  $Q$ .



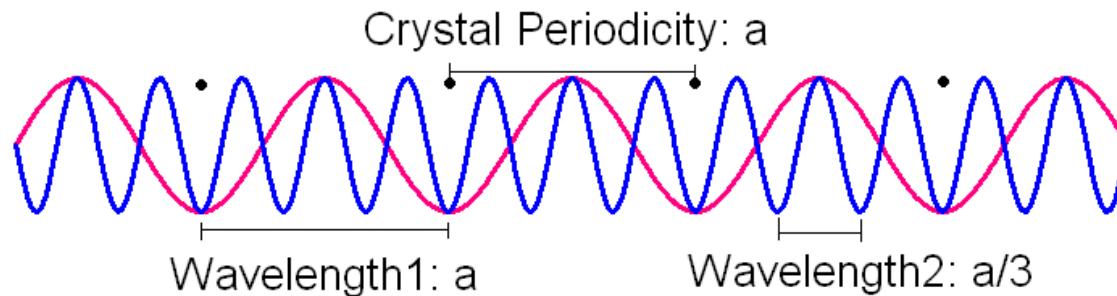
Distortion in the Fermi Surface can result in Peierls Instability and Kohn Anomaly in higher dimensions



# Commensurate Phase

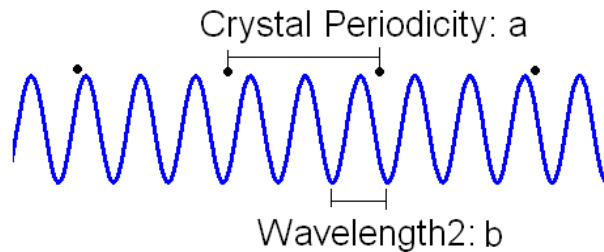
$$\Delta\rho(r) = \rho_0 \cos(Qr + \phi)$$

Since  $Q = 2k_F = \pi N/a \rightarrow \lambda = 2a/N$ , where the crystal periodicity is  $a$ , then if  $N$  is a rational number the CDW locks into the crystal.



Excitation energy necessary for conduction  
(semiconductor or Insulator)

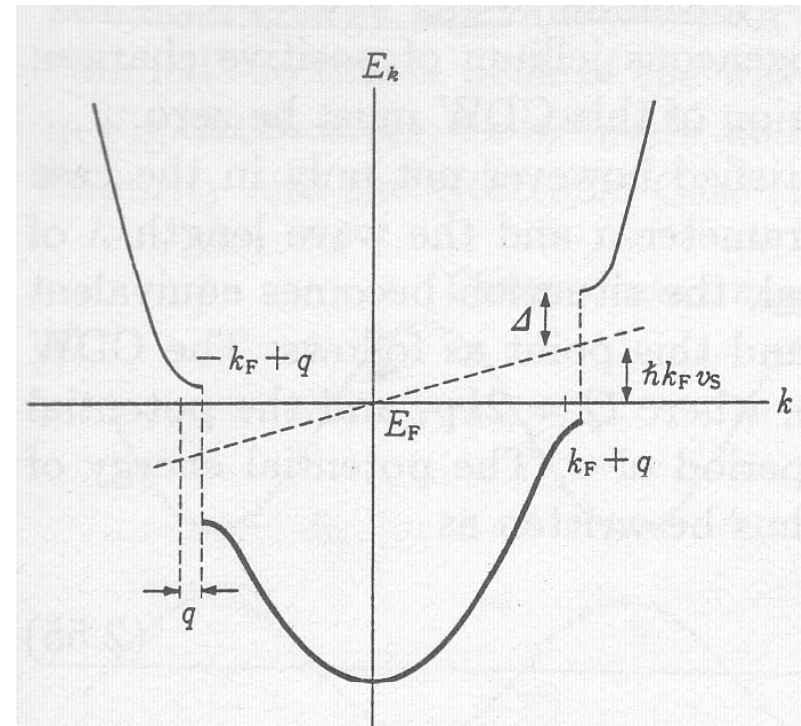
# Incommensurate Phase



$a/b$  irrational

$$\langle \Delta \rho(r) V(r) \rangle_r \approx 0$$

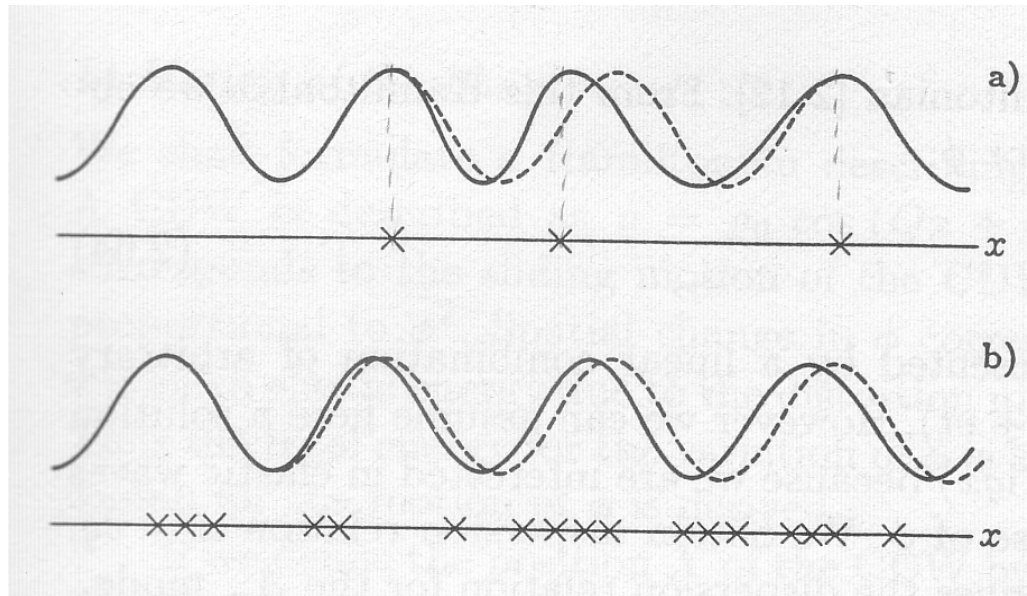
-Translational invariance



Ground State conducts  
without resistance when;

$$\frac{\hbar k_F}{\eta e} |J| < \Delta$$

# Impurity Pinning



Strong Pinning:

- CDW is phase locked at impurity sites

$$Qr_i + \phi(r_i) = c$$

Weak Pinning:

- CDW's phase has spatial variation

# References

- Kagoshima, S. et al. “One Dimensional Conductors.” Shokabo Publishing Co., Ltd., Tokyo 1982.
- Monceau, P. “Electronic Properties of Inorganic Quasi-One-Dimensional Compounds.” D. Reidel Publishing Company, Holland 1985.
- Grüner, G. et al. “Charge Density Waves in Solids.” Elsevier Science Publishers B.V, 1989.